A Sensitivity Analysis Procedure for Quality Function Deployment (QFD)

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Abstract

Quality Function Deployment (QFD) as part of the *Design for Six Sigma* (DFSS) methodology assists to design a new product or to redesign an existing product with respect to a set of potentially changing customer requirements. The goal of DFSS is that the novel product meets the stated requirements within a predefined Six Sigma quality level. If this aim is missed, sensitivity analyses may help to approach the aim. This article outlines how and with which means such analyses can be conducted.

1 Introduction

A prominent DFSS tool is the concept of *transfer functions*. For instance, transfer functions identify (highly) complex relationships between input parameters and the output of the product so that the quality of the product may be predicted, prior to a potentially cost intensive implementation of a prototype. Related to QFD, the *House of Quality* (HoQ) reflects a multidimensional transfer function.

Experience has shown that an initial HoQ does not provide an optimal or at least an acceptable solution. In this case the values of the input parameters, i.e., the values of the entries of the HoQ matrix, are modified, where such parameters are of interest which shift the result of the considered transfer function into the direction of a (nearly) optimal solution. How and to which extent each input parameter affects the result of the actual transfer function is the result of performing a *Global Sensitivity Analysis* (GSA), likewise another DFSS tool.

This article outlines how and by which means such a GSA can be conducted and what can be concluded from the result of the GSA.

2 DFSS Transfer Funktions

Transfer functions are of the general form

$$y = f(x_1, x_2, \ldots, x_n) = f(\boldsymbol{x}), \tag{1}$$

where $x_1, x_2, \ldots, x_n$ are termed the *controls* und y the *response* of f.

The response y is optimal, when y is equal to a predefined constant τ_y, the goal of f. If y is not optimal, i.e., if the *convergence gap* $|y - \tau_y| > 0$, then the controls $x_1, x_2, \ldots, x_n$ are modified by some strategy and y is re–calculated so that $|y - \tau_y|$ decreases.

In the context of QFD, transfer functions f are represented by an $m \times n$ matrix $\boldsymbol{A}$, i.e., $\boldsymbol{y} = \boldsymbol{A}(\boldsymbol{x})$ where $\boldsymbol{y}$ is a m–dimensional vector. Fehlmann and Kranich [2] point out, that solely the entries a_{ij} of $\boldsymbol{A}$ can be modified since in classical QFD the controls $\boldsymbol{x}$ are calculated by $\boldsymbol{x} = \boldsymbol{A}^T\boldsymbol{\tau_y}$ and after that the response $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} = \boldsymbol{A}\boldsymbol{A}^T\boldsymbol{\tau_y}$, where $\boldsymbol{\tau_y}$ is the prioritization vector of the customer requirements. Clearly, the response $\boldsymbol{y}$ is nearly optimal when the *convergence gap* $\|\boldsymbol{y} - \boldsymbol{\tau_y}\|_2 < \varepsilon$ where $\varepsilon > 0$ is a given threshold.

3 Global Sensitivity Analysis

The goal of a *Global Sensitivity Analysis* (GSA) is to gain an insight, how and to which extent a single control or a combination of controls affects the response $\boldsymbol{y}$ of a transfer funtion f. Among all GSA techniques the *variance–based procedures* play a prominent role, see, for instance, Saltelli et al. [3].

3.1 Variance–based Procedures

Considering the controls $x_2, x_2, \ldots, x_n$ in (1) as random variables $X_1, X_2, \ldots, X_n$ yields the transfer function

$$Y = f(X_1, X_2, \ldots, X_n) = f(\boldsymbol{X}). \tag{2}$$

Statements with respect to variance–based procedures are based on the *Law of Total Variance*, see, e.g., Saltelli et al. [3, p. 21]:

$$V(Y) = V_{X_i}(E_{\boldsymbol{X}_{\sim i}}(Y|X_i)) + E_{X_i}(V_{\boldsymbol{X}_{\sim i}}(Y|X_i)), \quad (3)$$

where $X_{\sim i}$ denotes all variables X_j with $j \neq i$. The term $V_{X_i}(E_{\boldsymbol{X}_{\sim i}}(Y|X_i))$ reflects the expected reduction in the variance of Y which would be achieved when a variable X_i is fixed at a specific value, whereas $E_{X_i}(V_{\boldsymbol{X}_{\sim i}}(Y|X_i))$ is equal to the expected residual variance with respect to Y when only the variable X_i is fixed.

In view of (3), the larger $V_{X_i}(E_{\boldsymbol{X}_{\sim i}}(Y|X_i))$, the smaller is $E_{X_i}(V_{\boldsymbol{X}_{\sim i}}(Y|X_i))$ and vice versa. A large variance $V_{X_i}(E_{\boldsymbol{X}_{\sim i}}(Y|X_i))$ means that X_i affects the response Y to a large extent. Hence, this variance is termed the *main effect* of X_i on Y and is measured by

$$S_i = \frac{V_{X_i}(E_{\boldsymbol{X}_{\sim i}}(Y|X_i))}{V(Y)}, \quad (4)$$

which is the measure of choice with respect to prioritizing of an (control) variable X_i.

Replacing X_i in (3) with the variables $\boldsymbol{X}_{\sim i}$ and vice versa yields

$$V(Y) = V_{\boldsymbol{X}_{\sim i}}(E_{X_i}(Y|\boldsymbol{X}_{\sim i})) + E_{\boldsymbol{X}_{\sim i}}(V_{X_i}(Y|\boldsymbol{X}_{\sim i})), \quad (5)$$

where the term $E_{\boldsymbol{X}_{\sim i}}(V_{X_i}(Y|\boldsymbol{X}_{\sim i}))$ is equal to the expected residual variance with respect to Y when the variables $X_{\sim i}$ having unknown true values are fixed. Therefore this term reflects the *total effect* X_i has on Y and is measured by

$$S_{T_i} = \frac{E_{\boldsymbol{X}_{\sim i}}(V_{X_i}(Y|\boldsymbol{X}_{\sim i}))}{V(Y)}, \quad (6)$$

which is the measure of choice with respect to fixing variables $\boldsymbol{X}_{\sim i}$.

Properties of S_i and S_{T_i} are, e.g., (a) A variable X_i is important when $S_{T_i} > 0.8$; it is important when $0.5 < S_{T_i} < 0.8$, unimportant, when $0.3 < S_{T_i} < 0.5$ and irrelevant when $S_{T_i} < 0.3$. (b) If $1 - \sum_i S_i > 0$, then interactions between variables exist. If $S_i < S_{T_i}$ then X_i interacts with the remaining $\boldsymbol{X}_{\sim i}$.

In general, it suffices to calculate estimators for S_i und S_{T_i}, see, e.g., Chan, Saltelli, and Tarantola [1].

3.2 The Winding Stairs (WS) Procedure

Chan, Saltelli, and Tarantola [1] describe the *Winding Stairs* (WS) procedure as a sampling technique which generates a sequence of samples of the variable X_i on the basis of an inital sample by modifying elements of the initial sample in a persisting cyclic order defined, e.g., by arranging component–wise the convergence gap $\|\boldsymbol{y} - \boldsymbol{\tau_y}\|_2$ in descending order, see Section 2.

The WS procedure is started by considering an initial sample $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ of the random variables $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)$. Subsequent samples $\boldsymbol{x_j}$ $(2 \leq j \leq n)$ are generated by modifying subsequently each single component of the prior generated sample $\boldsymbol{x_{j-1}}$. Each time a new sample $\boldsymbol{x_j}$ has been determined the corresponding response $y = f(\boldsymbol{x_j})$ is calculated. The first cycle of the WS procedure is complete when $\boldsymbol{x_n}$ and $y = f(\boldsymbol{x_n})$ has been computed. Subsequent cycles are handled in the same manner until the predefined maximum number m of cycles is reached.

In order to calculate estimators for S_i in (4) and S_{T_i} in (6) all responses $f(x_{\boldsymbol{k}})$ are organized in the $m \times n$ *Winding Stairs Matrix* (WS matrix):

$$\begin{pmatrix} f(\boldsymbol{x_1}) & f(\boldsymbol{x_2}) & \cdots & f(\boldsymbol{x_n}) \\ f(\boldsymbol{x_{n+1}}) & f(\boldsymbol{x_{n+2}}) & \cdots & f(\boldsymbol{x_{2n}}) \\ \vdots & \vdots & \vdots & \vdots \\ f(\boldsymbol{x_{(m-1)n+1}}) & f(\boldsymbol{x_{(m-1)n+2}}) & \cdots & f(\boldsymbol{x_{mn}}) \end{pmatrix} \quad (7)$$

Chan, Saltelli, and Tarantola [1] estimate the sensitivity indices S_i and S_{T_i} by using the WS matrix (7).

4 Outlook

This article outlines a sensitivity analysis procedure for QFD transfer functions. Implementation details and extensions of the presented method are subject of a forthcoming article.

References

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